Supersymmetry for Fermion Masses

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(Dated: January 4, 2014)

Abstract

It is proposed that supersymmetry (SUSY) maybe used to understand fermion mass hierarchies. A family symmetry Z_{3L} is introduced, which is the cyclic symmetry among the three generation $SU(2)$ doublets. SUSY breaks at a high energy scale $\sim 10^{11}$ GeV. The electroweak energy scale ~ 100 GeV is unnaturally small. No additional global symmetry, like the R-parity, is imposed. The Yukawa couplings and R-parity violating couplings all take their natural values which are $\mathcal{O}(10^0 - 10^{-2})$. Under the family symmetry, only the third generation charged fermions get their masses. This family symmetry is broken in the soft SUSY breaking terms which result in a hierarchical pattern of the fermion masses. It turns out that for the charged leptons, the τ mass is from the Higgs vacuum expectation value (VEV) and the sneutrino VEVs, the muon mass is due to the sneutrino VEVs, and the electron gains its mass due to both Z_{3L} and SUSY breaking. The large neutrino mixing are produced with neutralinos playing the partial role of right-handed neutrinos. $|V_{e3}|$ which is for $\nu_e - \nu_\tau$ mixing is expected to be about 0.1. For the quarks, the third generation masses are from the Higgs VEVs, the second generation masses are from quantum corrections, and the down quark mass due to the sneutrino VEVs. It explains m_c/m_s, m_s/m_e, $m_d > m_u$ and so on. Other aspects of the model are discussed.

PACS numbers: 12.15.Ff, 11.30.Pb, 11.30.Hv

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I. INTRODUCTION

In elementary particle physics, SUSY [1] was proposed for stabilizing the electroweak (EW) energy scale [2, 3] which is otherwise unnaturally small compared to the grand unification scale 3×10^{16} GeV [4, 5]. The study of the cosmological constant [6], however, suggests that unnaturalness of 10^{120} fine tuning might be just so from the anthropic point of view. It was argued that the string theory even supports the emergence of the anthropic landscape [7]. This led to a consideration of giving up naturalness of the EW scale [8, 9]. To keep gauge coupling constant unification and the dark matter, the so-called split SUSY [9, 10] was invented which has new features phenomenologically [11, 12, 13, 14, 15].

In Ref. [16], it was asked that if SUSY is not for stabilizing the EW scale, what else job does this beautifully mathematical physics do in particle physics, other than gauge coupling unification and the dark matter? We proposed to make use of SUSY to understand the lepton mass hierarchies. The flavor puzzle, namely the fermion masses, mixing and CP violation, in the Standard Model (SM) needs new physics to be understood [17]. The empirical fermion mass pattern is that the third generation is much heavier than the second generation which is also much heavier than the first. This may imply a family symmetry [18, 19, 20]. We first considered the charged leptons. By assuming a Z_3 cyclic symmetry among the $SU(2)$ doublets L_i ($i = 1, 2, 3$) of the three generations [19, 20], only the tau lepton gets mass, the muon and electron are still massless. The essential point is how the family symmetry breaks. Naively the symmetry breaking can be achieved by introducing family-dependent Higgs fields. We observed that SUSY naturally provides such Higgs-like fields, which are the scalar neutrinos. If the vacuum expectation values (VEVs) of the sneutrinos are non-vanishing, $v_i \neq 0$, the R-parity violating interactions $L_i L_j E_k^c$ [21, 22], with E_k^c denoting the anti-particle superfields of the SU(2) singlet leptons, contribute to the fermion masses, in addition to the Yukawa interactions. This is the origin of family symmetry breaking. The above idea has been proposed for some time [19, 20]. Because SUSY had been used to stabilize the EW scale, that idea suffered from severe constraints. For example, the τ-neutrino should be 10 MeV heavy [23]. It is a liberation if SUSY has nothing to do with the EW scale. Because the SUSY breaking scale is very high, the neutrinos are light. Furthermore, there is no need to introduce the R-parity or the baryon number as a symmetry.

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In this paper, after refining the lepton sector, we include discussion of the quark masses. To understand the large ratio of the top quark mass and the bottom quark mass, we assume that $\tan\beta$ is large. Numerically we make modification correspondingly. While the τ-lepton mass is from the down-type Higgs VEV ~ 10 GeV, the muon mass is due to v_i, $m_\mu \sim \lambda v_i$ with λ standing for the trilinear R-parity violation couplings. It is natural $\lambda \sim 10^{-1}$ like the Yukawa couplings for the τ mass. The muon mass tells us then $v_i \sim 1$ GeV. 1 GeV v_i's could induce a large lepton number violating effect, namely a large neutrino Majorana mass if the neutralinos are not heavy, due to $m_\nu \simeq (g_2 v_i)^2 / M_{\tilde{Z}}$, where g_2 is the $SU(2)_L$ gauge coupling constant, and $M_{\tilde{Z}}$ is the gaugino mass. When we take $M_{\tilde{Z}} \simeq 10^{11}$ GeV, the above formula can produce a neutrino mass needed to explain the solar neutrino problem.

This paper is organized as follows. In Sect. II, we will review, improve and expand the discussion of the lepton sector [16]. Quark sector is studied in Sect. III. In addition to the quark masses, mixing and CP violation are considered. Sect. IV gives the low energy effective theory. It will be easy and clear to discuss the neutrino masses and the lepton mixing in a separate section which is Sect. V. Sect. VI discusses some important and interesting aspects of the model. A summary is given in the final section.

II. LEPTONS

In our model the Z_{3L} family symmetry, that is invariance under $L_1 \rightarrow L_2 \rightarrow L_3 \rightarrow L_1$, mentioned in the beginning is assumed, which however is softly broken. The gauge symmetries and the matter contents in the full theory are the same as those in the SUSY SM. When the family symmetry is considered, the relevant kinetic terms should be written in a general form which keeps the symmetry,

$$
\mathcal{L} \supset \left(H_1^\dagger H_1 + H_2^\dagger H_2 + \alpha L_i^\dagger L_i + \beta(L_1^\dagger L_2 + L_2^\dagger L_3 + L_3^\dagger L_1 + h.c.) \right.
$$
$$
\left. + \frac{\gamma}{\sqrt{3}}(H_2^\dagger \sum_i L_i + h.c.) \right)\Bigg|_{\theta\theta\bar{\theta}\bar{\theta}} ,
$$

(1)

where H_1 and H_2 are the two Higgs doublets, α, β, γ are $O(1)$ coefficients. The case of that $\alpha = 1$ and $\beta = \gamma = 0$ is a special one of above expression. The superpotential is

$$
\mathcal{W} = \frac{\tilde{y}_j}{\sqrt{3}}(\sum_i L_i)H_2 E_j^c + \tilde{\lambda}_j(L_1 L_2 + L_2 L_3 + L_3 L_1)E_j^c + \tilde{\mu}H_1 H_2 + \tilde{\mu}'H_1 \sum_i L_i ,
$$

(2)

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where $\tilde{y}_j$'s and $\tilde{\lambda}_j$'s are the coupling constants. $\tilde{\mu}$ and $\tilde{\mu}'$ are mass terms. It is natural that their order closes to the scale of soft SUSY breaking masses. The Lagrangian of soft SUSY breaking masses is

$$
\begin{aligned}
\mathcal{L}_{soft1} &= M_{\tilde{W}}\tilde{W}\tilde{W} + M_{\tilde{Z}}\tilde{Z}\tilde{Z} \\
&\quad + m_h^2 h_1^\dagger h_1 + m_h^2 h_2^\dagger h_2 + m_{lL_{ij}}^2 \tilde{l}_i^\dagger \tilde{l}_j + m_{lR_{ij}}^2 \tilde{e}_i^* \tilde{e}_j \\
&\quad + (B_{\tilde{\mu}} h_1 h_2 + B_{\tilde{\mu}_i} h_1 \tilde{l}_i + m_i'^2 h_2^\dagger \tilde{l}_i + h.c.)\,,
\end{aligned}
\tag{3}
$$

where $\tilde{W}$ and $\tilde{Z}$ stand for the charged and neutral gauginos, respectively, h_1, h_2, $\tilde{l}_i$ and $\tilde{e}_i$ are the scalar components of H_1, H_2, L_i and E_i^c respectively. Note that explicitly breaking of Z_{3L} is introduced in the soft mass terms. The soft masses are assumed to be very large around a typical mass m_S. The trilinear soft terms should be also included,

$$
\mathcal{L}_{soft2} = \tilde{m}_{ij}\tilde{l}_i h_2 \tilde{e}_j + \tilde{m}_{ijk}\tilde{l}_i \tilde{l}_j \tilde{e}_k + h.c..
\tag{4}
$$

The mass coefficients which we denote generally as $\tilde{m}_S$ can be close to m_S.

The expression of the kinetic terms is not yet in the normalized canonical form. The standard form

$$
\mathcal{L} \supset \left(H_u^\dagger H_u + H_d'^\dagger H_d' + L_e^\dagger L_e + L_\mu^\dagger L_\mu + L_\tau'^\dagger L_\tau' \right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}
\tag{5}
$$

is achieved by the field re-definition:

$$
\begin{aligned}
H_u &= H_1\,, \\
H_d' &= c_1\left(H_2 + \frac{c_2}{\sqrt{3}}\sum_i L_i \right)\,, \\
L_\tau' &= c_1'\left(H_2 - \frac{c_2}{\sqrt{3}}\sum_i L_i \right)\,, \\
L_\mu &= \frac{c_3}{\sqrt{2}}(L_1 - L_2)\cos\theta + \frac{c_3}{\sqrt{6}}(L_1 + L_2 - 2L_3)\sin\theta\,, \\
L_e &= -\frac{c_3}{\sqrt{2}}(L_1 - L_2)\sin\theta + \frac{c_3}{\sqrt{6}}(L_1 + L_2 - 2L_3)\cos\theta\,,
\end{aligned}
\tag{6}
$$

where

$$
c_1 = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\gamma}{c_2}}\,,\quad c_2 = \sqrt{\alpha + 2\beta}\,,\quad c_3 = \sqrt{\alpha - \beta}\,,\quad c_1' = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\gamma}{c_2}}
\tag{7}
$$

and θ can not be determined until muon mass basis is fixed.

The superpotential is then

$$\mathcal{W} = \sqrt{\sum_j |y_j|^2} H'_d L'_\tau E^c_\tau + L_e L_\mu (\lambda_\tau E^c_\tau + \lambda_\mu E^c_\mu) + \mu H_u H'_d + \mu' H_u L'_\tau , \tag{8}$$

where

$$
\begin{aligned}
y_j &= \frac{2}{\sqrt{\alpha + 2\beta - \gamma^2}} \tilde{y}_j , &\quad \lambda_j &= -\frac{\sqrt{3}}{\alpha + \beta} \tilde{\lambda}_j , \\
\mu &= \frac{1}{2c_1} \left(\tilde{\mu} + \frac{\tilde{\mu}'}{c_2} \right) , &\quad \mu' &= \frac{1}{2c'_1} \left(\tilde{\mu} - \frac{\tilde{\mu}'}{c_2} \right) ,
\end{aligned}
\tag{9}
$$

E^c_τ is defined as

$$E^c_\tau = \frac{y_j}{y_\tau} E^c_j , \tag{10}$$

where $y_\tau \equiv \sqrt{\sum_j |y_j|^2}$. E^c_μ is orthogonal to E^c_τ, λ_τ and λ_μ are combinations of y_j's and λ_j's. Because of the Z_{3L} symmetry, the superpotential is without the field E^c_e which is orthogonal to both E^c_τ and E^c_μ.

To look at the fermion masses, we simply rotate the bilinear R-parity violating term away via the field re-definition,

$$H_d = \frac{1}{\bar{\mu}}(\mu H'_d + \mu' L'_\tau) , \quad L_\tau = \frac{1}{\bar{\mu}}(\mu' H'_d - \mu L_{\tau'}) , \tag{11}$$

where $\bar{\mu} \equiv \sqrt{\mu^2 + \mu'^2}$. It is trivial to see that the kinetic terms are diagonal in terms of H_d and L_τ. The superpotential is

$$\mathcal{W} = -y_\tau H_d L_\tau E^c_\tau + L_e L_\mu (\lambda_\tau E^c_\tau + \lambda_\mu E^c_\mu) + \bar{\mu} H_u H_d . \tag{12}$$

The Z_{3L} family symmetry keeps the trilinear R-parity violating terms invariant. As we have expected Higgs field H_d contributes to the tau mass only and the sneutrinos in L_e and L_μ contribute to the muon mass, after they get VEVs. The VEVs of L_e and L_μ imply the breaking of the Z_{3L} symmetry as can be seen explicitly from Eq. (6). The electron remains massless because of absence of the E^c_e field in $\mathcal{W}$. A hierarchy among charged leptons is obtained.

The breaking of the family symmetry originates from the soft SUSY masses. For simplicity and without losing generality, we assume that the soft terms in Eqs. (3) and (4) are rewritten as

$$
\begin{aligned}
\mathcal{L}_{soft} =&\ M_{\tilde{W}} \tilde{W}\tilde{W} + M_{\tilde{Z}} \tilde{Z}\tilde{Z} \\
&+ m^2_{h_u} h^\dagger_u h_u + m^2_{h_d} h^\dagger_d h_d + m^2_{h_d} \tilde{l}^\dagger_\alpha \tilde{l}_\alpha + m^2_{lR_{\alpha\beta}} \tilde{e}^*_\alpha \tilde{e}_\beta \\
&+ (B_\mu h_u h_d + B_{\mu_\alpha} h_u \tilde{l}_\alpha + \tilde{m}_{\alpha\beta} \tilde{l}_\alpha h_d \tilde{e}_\beta + \tilde{m}_{\alpha\beta\gamma} \tilde{l}_\alpha \tilde{l}_\beta \tilde{e}_\gamma + h.c.) ,
\end{aligned}
\tag{13}
$$

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where $\alpha = e, \mu, \tau$.

The key point of the form of the scalar masses lies in the $(h_u \ h_d^\dagger \ \tilde{l}_\alpha^\dagger)$ mass-squared matrix,

$$\mathcal{M}^{(h_u, h_d^\dagger, \tilde{l}_\alpha^\dagger)} = \begin{pmatrix} m_{h_u}^2 & B_\mu & B_{\mu e} & B_{\mu \mu} & B_{\mu \tau} \\ B_\mu & m_{h_d}^2 & 0 & 0 & 0 \\ B_{\mu e} & 0 & m_{h_d}^2 & 0 & 0 \\ B_{\mu \mu} & 0 & 0 & m_{h_d}^2 & 0 \\ B_{\mu \tau} & 0 & 0 & 0 & m_{h_d}^2 \end{pmatrix} \tag{14}$$

of which the eigenvalues are

$$\begin{aligned} M_1^2 &= \frac{m_{h_u}^2 + m_{h_d}^2}{2} - \sqrt{\left(\frac{m_{h_u}^2 - m_{h_d}^2}{2}\right)^2 + (B_\mu)^2 + \sum_\alpha (B_{\mu \alpha})^2} \\ M_2^2 &= \frac{m_{h_u}^2 + m_{h_d}^2}{2} + \sqrt{\left(\frac{m_{h_u}^2 - m_{h_d}^2}{2}\right)^2 + (B_\mu)^2 + \sum_\alpha (B_{\mu \alpha})^2} \\ M_3^2 &= M_4^2 = M_5^2 = m_{h_d}^2 \,. \end{aligned} \tag{15}$$

It is understood that $m_{h_u}^2$ and $m_{h_d}^2$ appeared in the matrix Eq. (14) denote the sum of the squared soft masses and the squared masses generated from the superpotential. $m_{h_u}^2$ can be negative. The analysis goes in the similar way as in Ref. [9]. By fine-tuning, $M_1^2 \sim -m_{EW}^2$, namely the EW symmetry breaking is achieved. The tuning is at the order of m_S^2 / m_{EW}^2.

In our case, in addition to the Higgs doublets, $\tilde{l}_\alpha$ fields also get VEVs,

$$v_u \neq 0, \quad v_d \neq 0, \quad v_{l_\alpha} \neq 0 \quad (\alpha = e, \ \mu, \ \tau). \tag{16}$$

The relative size of these values are determined by the soft mass parameters. It is easy to show from Eqs. (14) and (15) that $v_{l_\alpha}/v_d = B_{\mu \alpha}/B_\mu$ and $v_{l_\alpha}/v_{l_\beta} = B_{\mu \alpha}/B_{\mu \beta}$. It is therefore possible that hierarchies among v_u, v_d and v_{l_α} occur if there are hierarchies among the B_μ's. Note that the L_α numbers break explicitly in the soft mass terms, nonvanishing v_{l_α}'s do not result in any massless scalar. Because there is only one light Higgs doublet, the tree-level flavor changing neutral current (FCNC) does not appear. The hierarchical charged lepton mass pattern is obtained from Eq. (12) explicitly,

$$m_\tau \sim y_\tau v_d, \ m_\mu \sim \lambda_\mu \sqrt{v_{l_e}^2 + v_{l_\mu}^2}, \ m_e = 0 \,. \tag{17}$$

Numerically it is required that $v_d \sim 10$ GeV and $\sqrt{v_{l_e}^2 + v_{l_\mu}^2} \sim 1$ GeV. A careful analysis will be given in Sect. V.
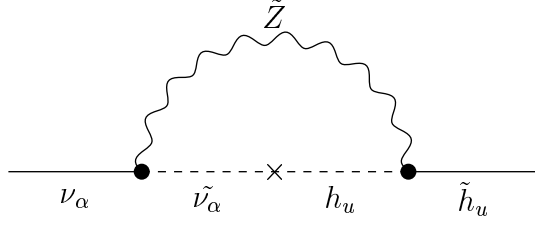
FIG. 1: B_{μ_α} induces a large lepton-Higgsino mixing via one loop.

It is important to note that masslessness of the electron is kept by SUSY. Generally, family symmetries keep the muon and electron massless. Once the family symmetry is broken, however, both muon and electron get their masses. And there is no reason to expect a hierarchy between the muon mass and the electron mass. In this model, it is the simplicity of the superpotential Eq. (8) that makes the electron massless even if the sneutrino VEVs are non-vanishing. The simplicity comes from SUSY. The non-vanishing electron mass is therefore due to SUSY breaking effects, as will be seen later.

If large v_{l_α}'s are safe should be studied. In addition, it should be also considered that huge B_{μ_α}'s induce large lepton-Higgsino mixing. The inducement happens at the loop-level through the gaugino exchange, as shown in Fig. 1 [23], $m_{\alpha h} = \dfrac{g_2^2 B_{\mu_\alpha}}{16\pi^2 M_{\tilde{Z}}}$ which is about $10^{-3} m_S$. By denoting $\tilde{h}$ as Higgsinos, the mass matrix of ν_α and the other neutralinos is given as

$$-i \left(\nu_e \ \nu_\mu \ \nu_\tau \ \tilde{h}_d^0 \ \tilde{h}_u^0 \ \tilde{Z} \right) \begin{pmatrix} 0 & 0 & 0 & 0 & m_{eh} & av_{l_e} \\ 0 & 0 & 0 & 0 & m_{\mu h} & av_{l_\mu} \\ 0 & 0 & 0 & 0 & m_{\tau h} & av_{l_\tau} \\ 0 & 0 & 0 & 0 & -\bar{\mu} & av_d \\ m_{eh} & m_{\mu h} & m_{\tau h} & -\bar{\mu} & 0 & -av_u \\ av_{l_e} & av_{l_\mu} & av_{l_\tau} & av_d & -av_u & M_{\tilde{Z}} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \\ \tilde{Z} \end{pmatrix}, \qquad (18)$$

where $a = (\dfrac{g_2^2 + g_1^2}{2})^{1/2}$ with g_1 being the SM $U(1)_Y$ coupling constant. We simply obtain the three large mass eigenvalues of the above mass matrix by reasonably taking v_{l_α}, v_d, $v_u \ll \bar{\mu}, M_{\tilde{Z}}$,

$$\Lambda_1 \simeq M_{\tilde{Z}}, \quad \Lambda_2 \simeq \bar{\mu}, \quad \Lambda_3 \simeq -\bar{\mu}. \qquad (19)$$

For the three light neutrinos, an interesting observation is that the mass matrix Eq. (18) is

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a realization of the see-saw mechanism [24]. In the above mass matrix, we denote M_R being the 3×3 lower-right submatrix, and m_{Dirac} the 3×3 upper-right submatrix. The heavy higgsinos and gauginos play the role of the right-handed neutrinos, a 3×3 light Majorana neutrino mass matrix is obtained as

$$\mathcal{M}_0^\nu \simeq -m_{\text{Dirac}} M_R^{-1} m_{\text{Dirac}}^T \,,$$

$$= -\frac{a^2}{M_{\tilde{Z}}} \begin{pmatrix} v_{l_e} v_{l_e} & v_{l_e} v_{l_\mu} & v_{l_e} v_{l_\tau} \\ v_{l_\mu} v_{l_e} & v_{l_\mu} v_{l_\mu} & v_{l_\mu} v_{l_\tau} \\ v_{l_\tau} v_{l_e} & v_{l_\tau} v_{l_\mu} & v_{l_\tau} v_{l_\tau} \end{pmatrix} \,. \tag{20}$$

The mass matrix is of rank 1. The nonvanishing mass is $m_\nu = \dfrac{a^2}{M_{\tilde{Z}}} v_{l_\alpha} v_{l_\alpha}$. It is very small $\sim 10^{-1} - 10^{-3}$ eV when $M_{\tilde{Z}} \sim 10^9 - 10^{11}$ GeV and $v_{l_\alpha} \sim (1-10)$ GeV. By introducing right-handed neutrinos, the neutrino sector has freedom to accommodate the realistic neutrino oscillation data.

We note that in the superpotential Eq. (12), the lepton number is violated. However, this violation is suppressed by gaugino and slepton masses, it has no observable effects at low energies. For example, the loop-induced electron-neutrino mass due to the R-parity violating trilinear interactions [19] is $m_{\nu_e} \simeq \dfrac{\lambda_\mu^2}{16\pi^2} \dfrac{\lambda_\mu v_d m_\mu}{m_S} \sim 10^{-5} - 10^{-6}$ eV. This is too small to be relevant to current neutrino physics.

The electron mass comes from the loop effects of Z_{3L} violation in the soft terms [19]. The soft breaking of Z_{3L} generates non-vanishing masses for the charged leptons through the one loop diagram Fig. 2, where χ and l, e^c denote the neutral gauginos and charged leptons. The mixing of the scalar leptons associated with different chiralities is due to the soft trilinear terms in Eq. (13), which is then about $y_\tau \tilde{m}_S v_d$. The exact formula for the one loop induced masses is

$$\delta M_{\alpha\beta}^l = \sum_\chi \frac{g_\chi^2}{16\pi^2} \frac{m_\chi}{m_\chi^2 - m_{\tilde{l}_\beta^c}^2} \left(\frac{m_\chi^2}{m_\chi^2 - m_{\tilde{l}_\alpha}^2} \ln \frac{m_{\tilde{l}_\alpha}^2}{m_\chi^2} + \frac{m_{\tilde{l}_\beta^c}^2}{m_{\tilde{l}_\alpha}^2 - m_{\tilde{l}_\beta^c}^2} \ln \frac{m_{\tilde{l}_\alpha}^2}{m_{\tilde{l}_\beta^c}^2} \right) y_\tau \tilde{m}_S v_d \,. \tag{21}$$

Approximately it is

$$\delta M_{\alpha\beta}^l \simeq \frac{\alpha}{\pi} \frac{y_\tau \tilde{m}_S v_d}{m_S} \,. \tag{22}$$

Taking $\tilde{m}_S / m_S \simeq 0.1$, $\delta M_{\alpha\beta}^l \sim \mathcal{O}(\text{MeV})$ which determines the electron mass. Note that the loop induced SUSY breaking effects are suppressed by the high SUSY breaking scale m_S in our case. This is different from the split SUSY case where $m_\chi / m_{\tilde{l}_\alpha^{(c)}} \to 0$.
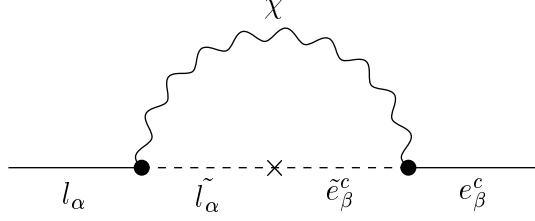
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FIG. 2: SUSY loop generation of the charged lepton masses. χ and l, e^c denote the neutral gauginos and charged leptons.

The loop in Fig. 2 does not cause any SUSY FCNC problem. When a photon line is attached to the internal lines, the amplitude of the FCNC process is suppressed by a factor v_d/m_S which in our scenario is unobservably small $\sim 10^{-9} - 10^{-10}$. Therefore the loop generates masses only. In effective theory language, it produces a purely ordinary SM Yukawa interaction. This point is different from the weak scale SUSY [25]. Actually, whenever the SUSY breaking scale is pushed arbitrarily high, while keeping a Higgs unnaturally light, the radiative fermion mass generation mechanism is viable.

III. QUARKS

Now let us come to the quark masses. Like that of the charged leptons, the quark masses also have three origins: the Higgs VEVs, the sneutrino VEV and the loop effects of the soft Z_{3L} violating terms. However, the roles of the sneutrino VEVs and the loop effects are switched [20]. The sneutrino VEVs contribute to the first generation quark masses, and the loop effects to the charm and strange quark masses. Under the family symmetry Z_{3L}, the three quark $SU(2)$ doublets Q_i are also cyclic. The Z_{3L} symmetric superpotential includes

$$
\begin{aligned}
\mathcal{W} \supset & \frac{y_j^u}{\sqrt{3}}(\sum_i Q_i)H_1 U_j^c + \frac{y_j^d}{\sqrt{3}}(\sum_i Q_i)H_2 D_j^c + \lambda'_{1j}\sum_i Q_i L_i D_j^c \\
& + \lambda'_{2j}(Q_1 L_2 + Q_2 L_3 + Q_3 L_1)D_j^c + \lambda'_{3j}(Q_1 L_3 + Q_2 L_1 + Q_3 L_2)D_j^c,
\end{aligned}
\tag{23}
$$

where U_i^c and D_i^c are the $SU(2)$ singlet superfields for the up- and down-type quarks, respectively. $y_j^{u(d)}$ and λ'_{ij} are coupling constants. The new soft terms are masses of the squarks and the trilinear terms corresponding to Eq. (23), but without Z_{3L} symmetry. The kinetic

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terms include

$$\mathcal{L} \supset \left[\alpha' Q_i^\dagger Q_i + \beta'(Q_1^\dagger Q_2 + Q_2^\dagger Q_3 + Q_3^\dagger Q_1 + h.c.)\right]\Big|_{\theta\theta\bar\theta\bar\theta}, \tag{24}$$

with α' and β' $\mathcal{O}(1)$ being coefficients. They are in the canonical form

$$\mathcal{L} \supset \left(Q_t^\dagger Q_t + Q_c^\dagger Q_c + Q_u^\dagger Q_u\right)\Big|_{\theta\theta\bar\theta\bar\theta} \tag{25}$$

by the following field redefinition,

$$
\begin{aligned}
Q_t &= \frac{\sqrt{\alpha' + 2\beta'}}{\sqrt{3}} \sum_i Q_i \\
Q_c &= \frac{c_3'}{\sqrt{2}}(Q_1 - Q_2)\cos\theta' + \frac{c_3'}{\sqrt{6}}(Q_1 + Q_2 - 2Q_3)\sin\theta' \\
Q_u &= -\frac{c_3'}{\sqrt{2}}(Q_1 - Q_2)\sin\theta' + \frac{c_3'}{\sqrt{6}}(Q_1 + Q_2 - 2Q_3)\cos\theta',
\end{aligned}
\tag{26}
$$

where $c_3' = \sqrt{\alpha' - \beta'}$, θ' is still an arbitrary parameter. The superpotential is then

$$
\begin{aligned}
\mathcal{W} \supset\ & y_t Q_t H_u U_t^c + y_b Q_t H_d D_b^c + Q_t L_\tau \sum_{\beta=b,s,d} \lambda'_{t\beta} D_\beta^c \\
& + (Q_c L_e - Q_u L_\mu) \sum_{\beta=b,s,d} \lambda'_{c\beta} D_\beta^c + (Q_u L_e + Q_c L_\mu) \sum_{\beta=b,s,d} \lambda'_{u\beta} D_\beta^c,
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
y_t &= \frac{1}{c_2'}\sqrt{\sum_i |y_i^u|^2}, \quad U_t^c = \frac{y_i^u}{\sqrt{\sum_j |y_j^u|^2}} U_i^c, \\
y_b &= \frac{1}{c_2'}\sqrt{\sum_i |\bar{y}_i^d|^2}, \quad D_b^c = \frac{\bar{y}_i^d}{\sqrt{\sum_j |\bar{y}_j^d|^2}} D_i^c, \\
\bar{y}_i^d &= \frac{1}{2\bar\mu}\left[\left(\frac{\mu}{c_1} + \frac{\mu'}{c_1'}\right)y_i^d + \frac{1}{c_1}\left(\frac{\mu}{c_2} - \frac{\mu'}{c_1'}\right)\sum_j \lambda'_{ji}\right], \\
\lambda'_{\alpha b} &= \frac{\sum_i \bar{\lambda}'_{\alpha i} \bar{y}_i^d}{\sqrt{\sum_j |\bar{y}_j^d|^2}} \quad \text{for } \alpha = t, c, u, \\
\bar{\lambda}'_{ti} &= \frac{1}{2c_2'\bar\mu}\left[\left(\frac{\mu'}{c_1} - \frac{\mu}{c_1'}\right)y_i^d + \frac{1}{c_1}\left(\frac{\mu'}{c_2} + \frac{\mu}{c_1'}\right)\sum_j \lambda'_{ji}\right], \\
\bar{\lambda}'_{ci} &= \frac{1}{2c_3 c_3'}[-(\sqrt{3}\cos\theta' - \sin\theta')(\lambda'_{1i} - \lambda'_{2i}) + (\sqrt{3}\cos\theta' + \sin\theta')(\lambda'_{1i} - \lambda'_{3i})], \\
\bar{\lambda}'_{ui} &= \frac{1}{2c_3 c_3'}[(\sqrt{3}\sin\theta' + \cos\theta')(\lambda'_{1i} - \lambda'_{2i}) - (\sqrt{3}\sin\theta' - \cos\theta')(\lambda'_{1i} - \lambda'_{3i})],
\end{aligned}
\tag{28}
$$

and $\lambda'_{\alpha s} D_s^c + \lambda'_{\alpha d} D_d^c$ is that of $\bar{\lambda}'_{\alpha i} D_i^c$, which is orthogonal to D_b^c.

Note that if we include $U_1^c \to U_2^c \to U_3^c \to U_1^c$ cyclic symmetry under the Z_{3L}, the above discussion does not change.

We see that if only the Higgs fields H_u and H_d get VEVs, the top quark and bottom quark are massive which will be denoted as m_0^t and m_0^b, respectively, and the other quarks are massless. Once the sneutrino $\tilde{\nu}_\alpha$ has a VEV, additional masses contribute to the down-type quarks. In the flavor basis given in Eq. (27), it is interesting to note that the R-parity violating couplings relevant only to the first two generations, which can be reduced to $\bar{\lambda}'_{cj}$ and $\bar{\lambda}'_{uj}$, do not involve any Yukawa coupling y_k^d. Therefore it will be natural if we take $\lambda'_{ij} \ll y_j^d$.

The soft breaking terms contribute masses to the quarks via loops [20, 26]. The way is the same as that producing the electron mass in Fig. 2, except for that the leptons are replaced by quarks, and neutralinos by the gluinos $\tilde{g}$,

$$\delta M_{\alpha\beta}^{u(d)} = \frac{\alpha_s}{\pi} \frac{2m_{\tilde{g}}}{m_{\tilde{g}}^2 - m_{\tilde{q}_\beta^c}^2} \left(\frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - m_{\tilde{q}_\alpha}^2} \ln \frac{m_{\tilde{q}_\alpha}^2}{m_{\tilde{g}}^2} + \frac{m_{\tilde{q}_\beta^c}^2}{m_{\tilde{q}_\alpha}^2 - m_{\tilde{q}_\beta^c}^2} \ln \frac{m_{\tilde{q}_\alpha}^2}{m_{\tilde{q}_\beta^c}^2} \right) (\tilde{m}_S^{u(d)})_{\alpha\beta} v_{u(d)} . \quad (29)$$

In order to make this contribution to be for the second generation only, we simply assume that the trilinear soft terms are independent on α (that is they keep the Z_{3L} symmetry) $(\tilde{m}_S)_{\alpha\beta} = \tilde{m}_{S_\beta}$, and that $m_{\tilde{q}^c} \ll m_{\tilde{q}} \sim m_{\tilde{g}}$,

$$\delta M_{\alpha\beta}^{u(d)} = \frac{2\alpha_s}{\pi} \frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - m_{\tilde{q}_\alpha}^2} \ln \frac{m_{\tilde{q}_\alpha}^2}{m_{\tilde{g}}^2} \frac{\tilde{m}_{S_\beta}^{u(d)} v_{u(d)}}{m_{\tilde{g}}} . \quad (30)$$

The point is that $\delta M_{\alpha\beta}$ is factorisable,

$$\delta M_{\alpha\beta}^q = f_\alpha^q \tilde{m}_\beta^q , \quad (31)$$

where q stands for u or d. f_α^q is a function of $m_{\tilde{g}}$ and $m_{\tilde{q}_\alpha}$, $f_\alpha^{u(d)} = \dfrac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - m_{\tilde{q}_\alpha}^2} \ln \dfrac{m_{\tilde{q}_\alpha}^2}{m_{\tilde{g}}^2}$, $\tilde{m}_\beta^{u(d)} = \dfrac{\tilde{m}_{S_\beta}^{u(d)} v_{u(d)}}{m_{\tilde{g}}}$. Neglecting $\langle \tilde{\nu}_\alpha \rangle$'s, the mass matrix is

$$\mathcal{M}_{\alpha\beta}^{u(d)} = \begin{pmatrix} f_{\alpha_1}^q \tilde{m}_{\beta_1}^q & f_{\alpha_1}^q \tilde{m}_{\beta_2}^q & f_{\alpha_1}^q \tilde{m}_{\beta_3}^q \\ f_{\alpha_2}^q \tilde{m}_{\beta_1}^q & f_{\alpha_2}^q \tilde{m}_{\beta_2}^q & f_{\alpha_2}^q \tilde{m}_{\beta_3}^q \\ f_{\alpha_3}^q \tilde{m}_{\beta_1}^q & f_{\alpha_3}^q \tilde{m}_{\beta_2}^q & f_{\alpha_3}^q \tilde{m}_{\beta_3}^q + m_0^{t(b)} \end{pmatrix} , \quad (32)$$

with α_i and β_i being (u, c, t) for q being u, and (d, s, b) for q being d. The mass matrix is of rank 2. Thus, at this stage, the second family quarks acquire masses. The first family remains massless. The above mass matrix determines the eigenvalues to the first order of

$f^q \tilde{m}^q / m_0^{t(b)}$,

$$m_{t(b)} \simeq m_0^{t(b)} \left[1 + \Re \frac{f_{\alpha_3}^{u(d)} \tilde{m}_{\beta_3}^{u(d)}}{m_0^{t(b)}} \right],$$

$$m_{c(s)} \simeq \sqrt{(|f_{\alpha_1}^{u(d)}|^2 + |f_{\alpha_2}^{u(d)}|^2)(|\tilde{m}_{\beta_1}^{u(d)}|^2 + |\tilde{m}_{\beta_2}^{u(d)}|^2)} \left[1 - \frac{2\Re(f_{\alpha_3}^{u(d)} \tilde{m}_{\beta_3}^{u(d)})}{m_0^{t(b)}} \right], \qquad (33)$$

$$m_{u(d)} = 0.$$

The order of magnitude of the masses of the second family can be understood naturally. From Eq. (29), we see that the charm quark to the strange quark mass ratio m_c/m_s is mainly determined by the ratio $(\tilde{m}_S^u v_u)/(\tilde{m}_S^d v_d)$ if there is no significant difference between the masses of the squarks with same chirality. The ratio $(\tilde{m}^u v_u)/(\tilde{m}^d v_d)$ can be $m_t/m_b \sim \mathcal{O}(10)$. Therefore the large ratio of m_c/m_s can be considered as a result of m_t/m_b. It should also be noted that in Eq. (29), we have neglected the other neutral gauginos, photino and Zino, because their effects are rather small compared with that of gluinos for the following two reasons. One is that α_s is large, $\alpha_s/\alpha \sim \mathcal{O}(10)$; another is that the number of gluinos is 8 which is also large. Hence the contribution of gluinos is nearly two orders of magnitude larger than that of photino or Zino. The radiative mass generation picture of quarks discussed above is consistent with that of leptons of Eq. (21) where it is the electron mass that is generated at the one-loop level by exchanging photino and Zino. The fact that the strange quark is two orders of magnitude heavier than the electron is thus explainable.

The quark mixing are then obtained. The mass-squared matrix $M^q M^{q\dagger}$ is diagonalized by

$$\begin{pmatrix} \frac{f_{\alpha_2}^{q*}}{\bar{f}^q} & -\frac{f_{\alpha_1}^{q*}}{\bar{f}^q} & 0 \\ \frac{f_{\alpha_1}^q}{\bar{f}^q} & \frac{f_{\alpha_2}^q}{\bar{f}^q} & \frac{\bar{f}^q \tilde{m}_{\beta_3}^q}{m_0^{t(b)}} \left[1 + \frac{f_{\alpha_3}^q}{a} \frac{|\tilde{m}_{\beta_1}^q|^2 + |\tilde{m}_{\beta_2}^q|^2 - |\tilde{m}_{\beta_3}^q|^2}{\tilde{m}_{\beta_3}^q} \right] \\ \frac{f_{\alpha_1}^q \tilde{m}_{\beta_i}^{q*} \tilde{m}_{\beta_i}^q}{m_0^{t(b)} \tilde{m}_{\beta_3}^q} \left(1 - \frac{f_{\alpha_3}^q \tilde{m}_{\beta_i}^{q*} \tilde{m}_{\beta_i}^q}{m_0^{t(b)} \tilde{m}_{\beta_3}^q} \right) & \frac{f_{\alpha_2}^q \tilde{m}_{\beta_i}^{q*} \tilde{m}_{\beta_i}^q}{m_0^{t(b)} \tilde{m}_{\beta_3}^q} \left(1 - \frac{f_{\alpha_3}^q \tilde{m}_{\beta_i}^{q*} \tilde{m}_{\beta_i}^q}{m_0^{t(b)} \tilde{m}_{\beta_3}^q} \right) & 1 \end{pmatrix},$$

$$(34)$$

where $\bar{f}^q \equiv \sqrt{|f_{\alpha_1}^q|^2 + |f_{\alpha_2}^q|^2}$ for q being u or d. The quark mixing matrix V_{CKM} is

$$\begin{pmatrix} \frac{f_{\alpha_1}^u f_{\alpha_1}^{d*} + f_{\alpha_2}^u f_{\alpha_2}^{d*}}{\bar{f}^u \bar{f}^d} & \frac{-f_{\alpha_1}^{u*} f_{\alpha_2}^{d*} + f_{\alpha_2}^{u*} f_{\alpha_1}^{d*}}{\bar{f}^u \bar{f}^d} & \frac{f_{\alpha_1}^{u*} f_{\alpha_2}^{d*} - f_{\alpha_2}^{u*} f_{\alpha_1}^{d*}}{\bar{f}^d} \frac{\tilde{m}_{\beta_i}^{u*} \tilde{m}_{\beta_i}^u}{m_0^t \tilde{m}_{\beta_3}^u} \\ \frac{f_{\alpha_1}^u f_{\alpha_2}^d - f_{\alpha_2}^u f_{\alpha_1}^d}{\bar{f}^u \bar{f}^d} & \frac{f_{\alpha_1}^{u*} f_{\alpha_1}^d + f_{\alpha_2}^{u*} f_{\alpha_2}^d}{\bar{f}^u \bar{f}^d} & -\frac{f_{\alpha_1}^{u*} f_{\alpha_1}^d + f_{\alpha_2}^{u*} f_{\alpha_2}^d}{\bar{f}^d} \left(\frac{\tilde{m}_{\beta_i}^{u*} \tilde{m}_{\beta_i}^u}{m_0^t \tilde{m}_{\beta_3}^u} \right) + \bar{f}^d \frac{\tilde{m}_{\beta_3}^{d*}}{m_0^b} \\ \frac{-f_{\alpha_1}^u f_{\alpha_2}^d + f_{\alpha_2}^u f_{\alpha_1}^d}{\bar{f}^u} \frac{\tilde{m}_{\beta_i}^{d*} \tilde{m}_{\beta_i}^d}{m_0^b \tilde{m}_{\beta_3}^d} & -\frac{f_{\alpha_1}^{u*} f_{\alpha_1}^d + f_{\alpha_2}^{u*} f_{\alpha_2}^d}{\bar{f}^u} \frac{\tilde{m}_{\beta_i}^{d*} \tilde{m}_{\beta_i}^d}{m_0^b \tilde{m}_{\beta_3}^d} + \bar{f}^u \frac{\tilde{m}_{\beta_3}^{u*}}{m_0^t} & 1 \end{pmatrix},$$

$$(35)$$

At first sight, we see that the Cabbibo angle [27] V_{us} and V_{cd} are not necessarily small. However, there are ways to achieve the smallness, for example, taking $f^q_{\alpha_1} = f^q_{\alpha_2}$. For simplicity, by taking $|f^q_{\alpha_1}| \simeq |f^q_{\alpha_2}| \simeq |f^q_{\alpha_3}|$ and $\tilde{m}^q_{\beta_1} \simeq \tilde{m}^q_{\beta_2} \simeq \tilde{m}^q_{\alpha_3}$, the quark mixing matrix can be consistent with experimental data. Furthermore, from Eq. (34) we explicitly obtain

$$V_{us} = -V_{cd}^*, \quad V_{ub} \sim -10^{-2}V_{us}, \quad V_{td} \sim 10^{-2}V_{us}, \quad \frac{V_{ub}}{V_{td}} \simeq \mathcal{O}(1), \quad V_{cb} \sim V_{ts} \sim \mathcal{O}(10^{-2}).$$
(36)

The first generation quarks get their masses from the sneutrino VEVs [20]. Coming back to Eq. (27), we see the quantities $\lambda'_{u\beta}v_{l_e}$ and $\lambda'_{c\beta}v_{l_\mu}$ have been implicitly taken to be smaller than the masses of the second family. However, they will produce a mass to the down quark of the first family. By assuming $\lambda'_{u\beta}$ and $\lambda'_{c\beta} \sim 10^{-2}$, its value can be several MeV numerically. In addition, the mass of the up quark of the first family cannot be produced in this way. This gives us an explanation of the fact that $m_d > m_u$, and even may bring us a solution to the strong CP problem [20]. We should note that in order to keep Eq. (29) factorisable, that is to make the trilinear soft terms to be the mass origin solely for the second generation, we have assumed $m^2_{\tilde{q}^c} \ll m^2_{\tilde{q}}$. A nonvanishing quantity $m^2_{\tilde{q}^c}/m^2_{\tilde{q}}$ contributes a mass to the first generation of quarks $\sim m_{c(s)}\frac{m^2_{\tilde{q}^c}}{m^2_{\tilde{q}}}$. For our above picture being valid, $m^2_{\tilde{q}^c}/m^2_{\tilde{q}}$ should be smaller than 10^{-3}.

CP violation originates from the SUSY soft breaking part. In general, there are several possible origins of CP violation within the framework of SUSY. The first one is the complex Yukawa couplings y_t and y_b in Eq. (27) from which, it is seen explicitly that their phases can be absorbed by the redefinition of the quark fields. The second possible origin is from the R-parity violating couplings λ''s. Their CP violation effect is suppressed by the heavy squarks. As for the small down quark mass terms $(\lambda'v_{l_\alpha})_{c\beta}$, and $(\lambda'v_{l_\alpha})_{u\beta}$, not only can most of their phases be rotated away, but also are they themselves very small numbers ($m_d \ll m_s$) in the mass matrix. Therefore, the R-parity violating terms are also not the source of the observed CP violation. In addition, due to the sneutrino VEV can be complex, the sneutrino exchange makes CP violating processes. However, these processes are suppressed by the heavy sneutrino. The third origin lies in the phases of the soft breaking terms. Such an origin is a specific feature of SUSY theories. These phases would in turn enter the quark mixing matrix through the radiative mass generation mechanism Eq. (29). We note that the experimental data of the neutron electric dipole moment does not require these phases

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to be small in this model, because its SUSY correction is suppressed by the heavy squarks. From Eq. (35), we see that the Kobayashi-Maskawa (KM) CP violation mechanism [28] can be realized. It is the third origin that is the reason CP violation occurs in our model.

IV. THE EFFECTIVE THEORY AND THE HIGGS MASS

The light Higgs is the following combination,

$$h = a_u h_u + a_d h_d^* + a_e \tilde{l}_e^* + a_\mu \tilde{l}_\mu^* + a_\tau \tilde{l}_\tau^* \,, \tag{37}$$

where

$$a_u = \frac{v_u}{v}, \quad a_d = \frac{v_d}{v}, \quad a_\alpha = \frac{v_{l_\alpha}}{v} \,, \tag{38}$$

where $v \equiv \sqrt{v_u^2 + v_d^2 + \sum_\alpha v_{l_\alpha}^2}$. The low energy effective theory is written as

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= y_{\tau\tau} l_\tau h^\dagger e_\tau^c + y_{\mu\mu} l_\mu h^\dagger e_\mu^c + y_{\mu\tau} l_\mu h^\dagger e_\tau^c + y_{e\tau} l_e h^\dagger e_\tau^c + y_{e\mu} l_e h^\dagger e_\mu^c \\
&\quad + y_e^{\alpha\beta} l_\alpha h^\dagger e_\beta^c + \frac{a^2 v_{l_\alpha} v_{l_\beta}}{M_{\tilde{Z}}} \nu_\alpha^{Tc} \nu_\beta \\
&\quad + y_{tt} q_t h t^c + y_{tb} q_t h^\dagger b^c + y_{cc}^{\alpha\beta} q_{c_\alpha} h c_\beta^c + y_{cs}^{\alpha\beta} q_{c_\alpha} h^\dagger s_\beta^c \\
&\quad + m^2 h^\dagger h - \frac{\lambda}{2} (h^\dagger h)^2 + h.c. \,,
\end{aligned}
\tag{39}
$$

where the effective Yukawa couplings are

$$
\begin{aligned}
y_{\tau\tau} &= y_\tau a_d \,, \; y_{\mu\mu} = \lambda_\mu a_e \,, \; y_{\mu\tau} = \lambda_\tau a_e \,, \; y_{e\tau} = \lambda_\tau a_\mu \,, \; y_{e\mu} = \lambda_\mu a_\mu \,, \; y_e^{\alpha\beta} = \frac{\delta M_{\alpha\beta}^l}{v} \\
y_{tt} &= y_t a_u \,, \; y_{tb} = y_b a_d \,, \; y_{cc}^{\alpha\beta} = \frac{\delta M_{\alpha\beta}^u}{v} \,, \; y_{cs}^{\alpha\beta} = \frac{\delta M_{\alpha\beta}^d}{v} \,,
\end{aligned}
\tag{40}
$$

and λ is determined by the gauge couplings,

$$\lambda = \frac{a^2}{2} \left(a_u^* a_u - a_d^* a_d - a_\alpha^* a_\alpha \right). \tag{41}$$

The above quantities are given at a high energy scale which is $m_S \simeq 10^{11}$ GeV. At the EW scale, their values can be calculated via the renormalization group method. It is expected that for most of them, the modification is not significant. We put such a systematic analysis for future works. Nevertheless, the Higgs mass should be discussed. As far as this point is concerned, our model is the same as that given in Ref. [13]. By taking $\tan\beta \sim m_t/m_b$, it was shown [13] that

$$m_h \simeq 145 \pm 7 \text{ GeV} \,, \tag{42}$$

where the uncertainty includes that of both m_t and α_s.

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V.   NEUTRINO OSCILLATION

Implications of the neutrino oscillations to this model need a more detailed study. The neutrino masses and lepton mixing deserve a separate section to be discussed. Eq. (20) would be a kind of democratic mass matrix [29] for neutrinos if $v_{l_e} \sim v_{l_\mu} \sim v_{l_\tau}$. The large mixing of the neutrinos would seem to be naturally accommodated. However, this would result in a large ν_e-ν_τ mixing. In addition, as we have mentioned, right-handed neutrinos are needed for the realistic neutrino oscillations.

A SM singlet superfield N is introduced. In the superpotential, the following terms should be added,

$$\mathcal{W} \supset \frac{\tilde{\kappa}_1}{\sqrt{3}} \sum_i L_i H_1 N + \tilde{M}_1^2 N + \tilde{M}_2 N N + \tilde{\kappa}_2 H_1 H_2 N + \tilde{\kappa}_3 N^3 \tag{43}$$

with $\tilde{\kappa}_i$'s being coupling constants and $\tilde{M}_i$'s masses supposed to be large. The linear term can be removed away via field redefinition. By defining $\bar{N} = N + n_0$ with n_0 being a constant field, we write

$$\tilde{M}_1^2 N + \tilde{M}_2 N^2 + \tilde{\kappa}_3 N^3 = \tilde{M} \bar{N}^2 + \tilde{\kappa}_3 \bar{N}^3 + C \,, \tag{44}$$

where $\tilde{M}$, n_0 and C satisfy

$$\begin{aligned}
\tilde{M} + 3\tilde{\kappa}_3 n_0 &= \tilde{M}_2 \,, \\
(2\tilde{M} + 3\tilde{\kappa}_3 n_0) n_0 &= \tilde{M}_1^2 \,, \\
C &= -\tilde{M} n_0^2 - \tilde{\kappa}_3 n_0^3 \,.
\end{aligned} \tag{45}$$

In terms of $\bar{N}$,

$$\mathcal{W} \supset \frac{\tilde{\kappa}_1}{\sqrt{3}} \sum_i L_i H_1 \bar{N} + \tilde{M} \bar{N} \bar{N} + \tilde{\kappa}_2 H_1 H_2 \bar{N} + \tilde{\kappa}_3 \bar{N}^3 \,, \tag{46}$$

where the constant C is omitted. Note that the field redefinition adds $\tilde{\kappa}_1 n_0 / \sqrt{3}$ and $\tilde{\kappa}_2 n_0$ to $\tilde{\mu}'$ and $\tilde{\mu}$ in Eq. (2), respectively. These are simply regarded as redefinition of $\tilde{\mu}'$ and $\tilde{\mu}$. Generally the corresponding soft terms can be written down. We assume that $\bar{N}$ does not develop any non-vanishing VEV. (There are other ways to eliminate the purely linear term of N with a large mass-squared coefficient. For example, N is assumed being charged under a larger gauge group than the SM [30]. The soft mass of N is assumed to be large enough that N has no any non-vanishing VEV.) Through the previous field redefinition, Eq. (46) then becomes

$$\mathcal{W} \supset \kappa_\tau H_u L_\tau \bar{N} + \tilde{M} \bar{N} \bar{N} + \kappa_d H_u H_d \bar{N} + \tilde{\kappa}_3 \bar{N}^3 \,, \tag{47}$$

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where

$$\kappa_\tau = \frac{1}{2\bar{\mu}}\left[\left(\frac{\mu'}{c_1} - \frac{\mu}{c_1'}\right)\tilde{\kappa}_2 - \left(\frac{\mu'}{c_2} + \frac{\mu}{c_1'}\right)\frac{\tilde{\kappa}_1}{c_1}\right],$$

$$\kappa_d = \frac{1}{2\bar{\mu}}\left[\left(\frac{\mu'}{c_1'} - \frac{\mu}{c_2}\right)\frac{\tilde{\kappa}_1}{c_1} + \left(\frac{\mu}{c_1} + \frac{\mu'}{c_1'}\right)\tilde{\kappa}_2\right].$$

(48)

The last two terms in Eq. (47) do not play important roles to our analysis. The first two terms contribute a mass term to the neutrino mass matrix by the seesaw mechanism,

$$m_{\nu_\tau\nu_\tau} \simeq -\frac{(\kappa_\tau v_u)^2}{\tilde{M}}.$$

(49)

From Eq. (20) or (39) and the above mass term, the full neutrino mass matrix is

$$\mathcal{M}^\nu = -\frac{a^2}{M_{\tilde{Z}}}\begin{pmatrix} v_{l_e}v_{l_e} & v_{l_e}v_{l_\mu} & v_{l_e}v_{l_\tau} \\ v_{l_\mu}v_{l_e} & v_{l_\mu}v_{l_\mu} & v_{l_\mu}v_{l_\tau} \\ v_{l_\tau}v_{l_e} & v_{l_\tau}v_{l_\mu} & v_{l_\tau}v_{l_\tau} + x \end{pmatrix}$$

(50)

with x being $\frac{M_{\tilde{Z}}}{\tilde{M}}\left(\frac{\kappa_\tau v_u}{a}\right)^2$. We find that realistic lepton physics can be obtained by taking $x \sim v_{l_\tau}v_{l_\tau} \gg v_{l_e}^2 + v_{l_\mu}^2 \sim 1~\text{GeV}^2$. The eigen values are

$$m_{\nu_3} \simeq \frac{a^2}{M_{\tilde{Z}}}v_{l_\tau}^2 + \frac{(\kappa_\tau v_u)^2}{\tilde{M}},$$

$$m_{\nu_2} \simeq \frac{a^2}{M_{\tilde{Z}}}(v_{l_e}^2 + v_{l_\mu}^2)\frac{x}{x + v_{l_\tau}^2},$$

$$m_{\nu_1} = 0.$$

(51)

The solar neutrino problem requires that $m_{\nu_2} \simeq (10^{-2} - 10^{-3})~\text{eV}$ which is achieved when $M_{\tilde{Z}} \sim 10^{11}~\text{GeV}$. Suppose $v_{l_\tau} \sim 10~\text{GeV}$, the atmospheric neutrino problem requires a certain cancellation between the terms $\frac{a^2}{M_{\tilde{Z}}}v_{l_\tau}^2$ and $\frac{(\kappa_\tau v_u)^2}{\tilde{M}}$, in order to make $m_{\nu_3} \sim 10^{-1} - 10^{-2}$. The mass matrix Eq. (50) is diagonalized by U_ν,

$$U_\nu = \begin{pmatrix} \dfrac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_e}}{\sqrt{v_{l_\alpha}v_{l_\alpha}}} \\ \dfrac{-v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_\mu}}{\sqrt{v_{l_\alpha}v_{l_\alpha}}} \\ 0 & -\dfrac{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}}{v_{l_\tau} + x/v_{l_\tau}} & \dfrac{v_{l_\tau}}{\sqrt{v_{l_\alpha}v_{l_\alpha}}} \end{pmatrix}.$$

(52)

Going back to the charged lepton masses, the mass matrix is seen from Eq. (12) or (39)

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- (40),

$$\mathcal{M}^l = \begin{pmatrix} 0 & \lambda_\mu v_{l_\mu} & \lambda_\tau v_{l_\mu} \\ 0 & \lambda_\mu v_{l_e} & \lambda_\tau v_{l_e} \\ 0 & 0 & y_\tau v_d \end{pmatrix}. \tag{53}$$

To obtain agreement with the experimental data, we assume that $y_\tau v_d \sim \lambda_\tau v_{l_\mu} \sim \lambda_\tau v_{l_e} \sim 1$ GeV. In this case, the mass eigenvalues are

$$\begin{aligned} m_\tau &\simeq \sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}, \\ m_\mu &\simeq |\lambda_\mu| \sqrt{v_{l_e}^2 + v_{l_\mu}^2} \frac{y_\tau v_d}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}}, \\ m_e &= 0. \end{aligned} \tag{54}$$

The mass-squared matrix $\mathcal{M}^l \mathcal{M}^{l\dagger}$ is diagonalized by U_l,

$$U_l = \begin{pmatrix} \dfrac{-v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{y_\tau v_d}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} & \dfrac{\lambda_\tau v_{l_\mu}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} \\ \dfrac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \dfrac{y_\tau v_d}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} & \dfrac{\lambda_\tau v_{l_e}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} \\ 0 & \dfrac{-\lambda_\tau^* \sqrt{v_{l_e}^2 + v_{l_\mu}^2}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} & \dfrac{y_\tau v_d}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}} \end{pmatrix}. \tag{55}$$

The lepton mixing matrix is $V \equiv U_l^\dagger U_\nu$. The ν_e-ν_μ mixing is

$$|V_{e2}| = \frac{v_{l_\mu}^2 - v_{l_e}^2}{v_{l_e}^2 + v_{l_\mu}^2}. \tag{56}$$

It is $\mathcal{O}(1)$ by taking $v_{l_e} \sim v_{l_\mu}$. The ν_μ-ν_τ mixing is

$$|V_{\mu3}| \simeq \frac{|\lambda_\tau| \sqrt{v_{l_e}^2 + v_{l_\mu}^2}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}}. \tag{57}$$

A maximal mixing is approached if $y_\tau v_d$ is getting equal to $|\lambda_\tau| \sqrt{v_{l_e}^2 + v_{l_\mu}^2}$. (That is the reason we have assumed a large $\lambda_\tau \sim \mathcal{O}(1)$.) The current data show that $|V_{\mu3}|$ can be as small as 0.6 at the 99% C.L. [31]. Finally the ν_e-ν_τ mixing is

$$|V_{e3}| \simeq \frac{v_{l_\mu}^2 - v_{l_e}^2}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2} \, v_\tau}. \tag{58}$$

It is small ~ 0.1 if $\sqrt{v_{l_e}^2 + v_{l_\mu}^2}/v_\tau \sim 0.1$. A generic expectation of $|V_{e3}|$ is around 0.1 which is near to its experimental limit $|V_{e3}| < 0.17$ [31].

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VI. DISCUSSIONS

Some important aspects of our model should be discussed. In this framework, we do not need to introduce any extra symmetry, like the R-parity, or the baryon number, to forbid the proton decay. The interactions of baryon number violation should be included in the superpotential in principle,

$$\mathcal{W} \supset \lambda''_{ijk} U_i^c D_j^c D_k^c \,, \tag{59}$$

with λ''_{ijk} being coupling constants. Here the analysis is essentially the same as that of R-parity violation in split SUSY [15]. Because the sparticles are very heavy, they suppress baryon number and some lepton number violating processes to be unobservable, despite that the coupling constants of the baryon and lepton number violating interactions are large. For example, in the $\tau \to \mu^+ \mu^- e$ decay which occurs at tree level, the branching ratio is about $\sim 10^{-23}$ if $m_S \sim 10^{11}$ GeV. The proton decay measurements constraint [32] $\lambda' \lambda'' \leq 10^{-27} \dfrac{m_S^2}{(100 \text{ GeV})^2}$. When $m_S \sim 10^{11}$ GeV and $\lambda' \sim 10^{-2}$, λ'' is required to be smaller than 10^{-7}.

As we have mentioned in Sect. III, U_i^c's can compose a nontrivial representation of Z_{3L} symmetry. In this case, the baryon number violating terms in Eq. (59) is written as

$$\mathcal{W} \supset \sqrt{3} \lambda''_{tjk} U_t^c D_j^c D_k^c \,, \tag{60}$$

where $U_t^c = \dfrac{1}{\sqrt{3}}(U_1^c + U_2^c + U_3^c)$. This interaction does not lead to proton decays in the massless up-quark case, and λ'' can be $\mathcal{O}(1)$ numerically. Consider the case of a nonvanishing up quark mass, the proton decay rate is suppressed by the up-quark and top-quark mass ratio which is $(m_u/m_t)^2 \sim 10^{-10}$, and λ'' can be 10^{-2}.

From the EW symmetry breaking point of view, this model is nothing but a fine-tuned minimal SUSY SM with a high SUSY breaking scale. In spite of losing naturalness of the EW energy scale, the radiative breaking mechanism for the EW gauge symmetry [3, 33] may still remain in this model, because the situation is similar to that in the minimal SUSY SM. This SUSY model does not suffer from the so-called μ-problem [34]. Throughout the analysis, $\tilde{\mu}^{(\prime)}$ in Eq. (2) are not necessarily required to have the same order as that of the soft masses. They can be several orders smaller than the soft masses. Furthermore even the soft masses are not required to be at the same order. Because these masses are much larger than the EW scale, their differences do not cause any inconsistency phenomenologically. It
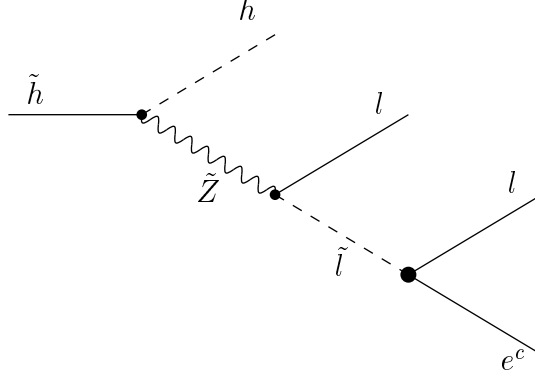
FIG. 3: A Higgsino decays to a Higgs and a virtual gaugino which further goes into a lepton and a virtual slepton, the slepton decays to a lepton pair via R-parity violating interaction.

seems that the unification of the gauge coupling constants is lost. There are arguments [12] that the unification can be still true in the case of high scale SUSY breaking. Nevertheless, the general trend of approximate unification is still there. This model does not suffer from the second fine-tuning of the split SUSY [14], because SUSY has no huge split in this model.

It would be difficult for experiments to verify the model directly, except for the 145 GeV m_h and the $\mathcal{O}(0.1)$ θ_{13}. At low energies, the model is basically the same as the SM. One essential feature of this model is that the unnaturally light Higgs has a component of a slepton. Related to this point, the model allows for relatively long-lived Higgsinos. We may consider a case where their masses are lower than m_S. If they are loop induced, the Higgsino masses are thousand times smaller than m_S. A Higgsino decays to a Higgs and a virtual gaugino which further goes into a lepton and a virtual slepton, the slepton decays to a lepton pair via R-parity violating interaction (Fig. 3). Because this four body decay is suppressed by the R-parity violating coupling and double suppressed by m_S, a 10^8 GeV heavy Higgsino has a lifetime of 10^{-5} sec.

The cosmological and astrophysical implications should be studied in future works. CP violation at high energies $\geq m_S$ has various origins, like the phases in soft SUSY breaking terms, in R-parity violating couplings, and in the sneutrino VEV. They might be the root of the matter-antimatter asymmetry of the Universe. However at the EW scale, many of these sources are suppressed. What left is reduced to the KM mechanism. On the other hand, we have noted that there is no natural dark matter candidate in this model. Existence

of the dark matter implies that the dark matter is really dark, namely it only has gravity interaction with the ordinary matter.

It should be studied in future works how the soft SUSY breaking terms also break the Z_{3L} family symmetry. Currently we understand this as follows. The gravity breaks any global symmetry like our family symmetry. The superpotential is for particle physics, which decouples from the gravity, and therefore keeps the family symmetry. Whereas the soft SUSY breaking terms are due to gravitational interaction, they may violate the family symmetry explicitly. This model would prefer a large ν_e-ν_τ mixing, if we had not looked at the experimental data for the leptons. A large $\lambda_\tau \sim 1$ is assumed in order to fit the data. In addition, a special structure of the soft breaking terms of the squark was assumed to produce the second and first generation quark mass hierarchy. The problem that how natural these assumption are also needs further studies.

There are various particle physics models which do not care the naturalness problem of the SM. This model seems to be one of them. However, this model is unique in the sense that it makes use of a mechanism which used to be directly for naturalness of the SM. In other words, while many other models can be naturalized after their SUSY extension, this model is intrinsically unnatural. Unnaturalness is understood by the anthropic principle. One meaningful question is then that why are not the SM Yukawa couplings anthropically determined? We answer this question from the following aspects. One is that although the electron mass might be understood from the anthropic point of view, it is hard to say all the Yukawa couplings including the neutrino masses being anthropically determined. Theoretically we have given up naturalness in the t'Hooft sense. Rather, Dirac naturalness has been emphasized in considering the fermion mass hierarchies. Whether such an effort can be justified should be studied in the more fundamental theory.

Finally we make a remark on the possibility of lowering the SUSY breaking scale from 10^{11} GeV to 1 TeV. This work essentially has made use of SUSY to understand fermion masses. The SUSY breaking scale is fixed by the neutrino masses implied by the neutrino oscillations. It would be much more interesting if SUSY also plays the role of stabilizing the EW scale [2]. We note that it is still possible that the τ-neutrino mass is about 10 MeV [23, 35, 36] (in this case the atmospheric neutrino problem is due to $\nu_\mu - \nu_{\text{sterile}}$ oscillation which is also yet ruled out experimentally), then the SUSY breaking scale will be about a few TeV. In that case the EW energy scale will be marginally natural.

VII. SUMMARY

If SUSY is not for stabilizing the EW energy scale, what is it used for in particle physics? In this paper, motivated by our previous works [16, 19, 20], we have proposed that SUSY is for flavor problems. A family symmetry Z_{3L}, which is the cyclic symmetry among the three generation $SU(2)_L$ doublets, is introduced. No additional global symmetry, like the R-parity is imposed. SUSY breaks at a high scale $\sim 10^{11}$ GeV. The EW energy scale ~ 100 GeV is unnaturally small from the point of view of the field theory. Under the family symmetry, only the third generation fermions get to be massive after EW symmetry breaking. This family symmetry is broken by soft SUSY breaking terms. These terms contribute masses via loops to the second generation quarks and the electron. Furthermore they induce sneutrino VEVs which result in the masses of the muon and the down quark. The neutrino large mixing can be obtained. The KM mechanism of CP violation is realized at low energies. A hierarchical pattern of the lepton and quark masses are obtained. The Higgs mass of this model is about 145 GeV. This point can be tested in the future experiments at Tevatron and LHC. It is expected that ν_e-ν_τ mixing is near to its experimental limit.

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